PHASE INTERACTION IN CONCENTRATED DISPERSE SYSTEMS

Yu. A. Buevich

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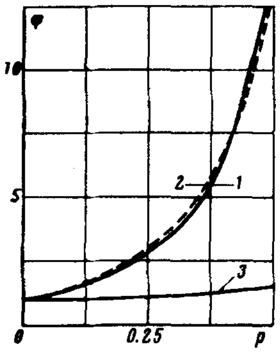
In maximally diluted disperse systems, each particle (solid, liquid or gaseous) may be regarded as being located in the unperturbed hydrodynamic field of the dispersion medium. In this approximation the force of viscous interphase interaction is additive with respect to the particles and in the simplest case represents the Stokes force multiplied by the number of particles in a particular volume of mixture. An enormous number of experimental and theoretical studies has been devoted to the interaction of particles moving relative to a fluid phase (e.g., under the influence of gravity). Most of these consider only a finite number of adjacent particles falling in a fluid and the hydrodynamic body forces developed in this system. In this case it turns out that the effect of these forces is to produce an increase in the rate of fall of each individual particle as compared with the Stokes velocity.



At the same time, in extensive disperse systems precisely the opposite effect is observed, which cannot be explained solely in terms of the body forces between pairs of particles. For example, to maintain solid particles in a fluidized state it is necessary for the fluid phase to move relative to these particles with an average velocity which is often an order of magnitude smaller than the Stokes velocity. Below, a simple expression, which is found to be in satisfactory agreement with the experimental facts, is proposed for relating the phase slip velocity and the viscous force acting on a single particle of the disperse system.

We shall consider a suspension of spherical formations of radius a in a fluid medium, located, for example, in a gravity field. For a high volume concentration of the spheres we may assume that the influence of each of them on the flow of the fluid phase extends only a finite distance, depending on the concentration, from the sphere. In other words, in a concentrated disperse system there is a certain effective screening of the long-range hydrodynamic interaction, and the flow perturbation due to the particles is essentially confined to a certain limited volume adjacent to each particle. Such models have previously been used in evaluating the viscosity of suspensions in the theory of aerosols, etc. (see, for example, [1-3]). The exact determination of the mean of these volumes, its size and shape, is impossible in the absence of details on the microstructure of the system and the forces of interparticle interaction, However, in the first approximation this volume can be described with the help of a "cellular model" of the system similar to the corresponding model in the kinetic theory of dense gases. Thus, we may assume that on the average each particle corresponds to a spherical cell concentric with the particle and having a radius $a' > a$, while perturbations of the flow outside this cell do not affect the flow of fluid inside it, and conversely. Mathematically, this is expressed by specifying at the surface of such a "sphere of influence" the boundary condition that the radial component of the perturbation velocity, introduced by the particle in question inside the sphere, vanishes.

We shall estimate the quantity a'. The specific volume of an individual particle $\tau = 1/n = \theta/\rho$, where θ is the volume of the particle, and n and ρ are the number and volume concentrations of the particles. Obviously,

$$a' = ka\rho^{-1/3}. \tag{1}$$

Here k is a parameter close to unity; generally speaking, k may be weakly dependent on ρ.

We solve the problem of the constricted flow around a particle by the method used in [4]. Outside the particle the linearized Stokes equations have the form

$$\mu\Delta\mathbf{v} - \nabla p = 0, \qquad \nabla\mathbf{v} = 0,$$

whence it follows that

$$\Delta\,[\nabla\times\mathbf{v}] = 0. \tag{2}$$

Analogous equations apply inside the sphere of radius a. For definiteness, we shall henceforth denote all quantities relating to the internal flow with a prime.

The divergence of the vector v is identically equal to zero; therefore v may be expressed in the form of the curl of a certain axial vector w, which must be linearly dependent on the average interphase slip velocity vector u. Such a vector can be uniquely constructed, i.e., we obtain

$$\mathbf{v} = \nabla\times[\nabla\times(f\mathbf{u})]. \tag{3}$$

Here f is some scalar function of the distance from the center of the particle r. From (2) and (3) we have

$$\Delta\,[\nabla\times\mathbf{v}] = \Delta\,(\nabla\nabla - \Delta)\,[\nabla\times f\mathbf{u}] = -\Delta^2\,[\nabla\times f\mathbf{u}] = 0.$$

Since the vector u is constant and arbitrary, we thus obtain

$$\Delta^2(\nabla f) = 0, \quad \Delta^2 f = \mathrm{const}.$$

The general solution of this equation, which depends only on r, can be represented in the form

$$f = \alpha r^4 + \beta r^2 + \gamma r + \delta r^{-1}.$$

Evaluating, we obtain for the pressures and the velocity components inside and outside the sphere in a spherical coordinate system introduced in the usual way

$$v_r = \left(\frac{A}{r^3} + \frac{B}{r} + Cr^2 + u\right)\cos\theta, \qquad v_r' = (cr^2 + d)\cos\theta,$$

$$v_\theta = \left(\frac{A}{2r^3} - \frac{B}{2r} - 2Cr^2 - u\right)\sin\theta, \qquad v_\theta' = (-2cr^2 - d)\sin\theta,$$

$$p = \mu\,(B/r^2 + 10Cr)\cos\theta, \quad p' = \mu'\,(10cr + e)\cos\theta. \tag{4}$$

Here A, B, C, c, d, e are constants. The boundary conditions have the form

$$v_r = v_r' = 0, \qquad v_\theta = v_\theta',$$

$$r = a, \qquad v_r = u\cos\theta, \qquad r = a',$$

$$-p + 2\mu\frac{\partial v_r}{\partial r} = -p' + 2\mu'\frac{\partial v_r'}{\partial r}, \qquad r = a,$$

$$\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right) = \mu'\left(\frac{1}{r}\frac{\partial v_r'}{\partial\theta} + \frac{\partial v_\theta'}{\partial r} - \frac{v_\theta'}{r}\right). \tag{5}$$

After evaluation of the constants from conditions (5) and their substitution in (4), we get for the components of the viscous stress tensor outside the sphere:

$$\sigma_{rr} = \frac{3\mu u}{aP(\xi)}\left[1+\frac{2}{\varkappa}-10\xi^3+\left(4-\frac{2}{\varkappa}\right)\xi^5\right]\cos\theta,$$

$$\sigma_{r\theta} = -\frac{3\mu u}{aP(\xi)}(1+5\xi^3-\xi^5)\sin\theta,$$

$$P(\xi) = 2\left(1+\frac{1}{\varkappa}\right)-5\xi^3+\left(3-\frac{2}{\varkappa}\right)\xi^5,$$

$$\xi=\frac{a}{a'}, \qquad \varkappa=\frac{\mu'}{\mu}.$$

Evaluation of the force F acting on a single particle gives

$$F = 4\pi\mu a u\lambda(1+G(\xi,\varkappa)), \qquad \lambda=\frac{3\mu'+2\mu}{2(\mu'+\mu)},$$

$$G(\xi,\varkappa) = \frac{\xi^3}{P(\xi)}\left\{5-\left[3-\frac{2}{\varkappa}+\frac{2}{\lambda}\left(1-\frac{1}{\varkappa}\right)\right]\xi^2\right\}. \quad (6)$$

Let us consider the limiting cases. When $\varkappa \to \infty$, $\lambda \to 3/2$ (solid particles)

$$F = 6\pi\mu a u(1+{}^5/_3 M(\xi)), \qquad M(\xi)=\frac{\xi^3(3-\xi^2)}{2-5\xi^3+3\xi^5}. \quad (7)$$

As $\varkappa \to 0$, $\lambda \to 1$ (gas bubbles in a viscous liquid) we have

$$F = 4\pi\mu a u(1+2N(\xi)), \qquad N(\xi)=\xi^5(1-\xi^5)^{-1}. \quad (8)$$

It is easy to see that as $\xi \to 0$ from (6) we get the familiar Rybchinskii-Hadamard formula, and from (7) or (8) its particular cases. From (6) it is easy to see that F goes to infinity as $\xi \sim \rho^{1/3}$, as it should; in reality, of course, the maximum possible values of ρ are equal to those for close packing (cubic, hexagonal), i.e., are less than unity. The expressions obtained are valid only if the particles are spherical or nearly spherical. This assumption is valid for droplets and bubbles if the surface tension at the phase interface is sufficiently large, or if the linear dimensions of the particles are sufficiently small.

It is of some interest to compare the results of this study with the experimental data. It is known [5] that in the region of small Reynolds numbers the viscous drag of a fluidized bed is satisfactorily described by a relation of the type (7), with $1+(5M/3)$ replaced by the quantity $\psi = (1-\rho)^{-3.75}$. The results of a numerical calculation of the functions $\varphi = 1 + 5M/3$ and ξ, computed from (1) for $k \approx 1.1$, and ψ are presented in the figure (curves 1 and 2, respectively). It is clear that the correspondence is very good. The same figure shows the function $1+2N$ (curve 3). A comparison of curves 1 and 3 yields the interesting conclusion that the difference between the forces of viscous interaction of solid and gaseous dispersed phases with the liquid phase in concentrated disperse systems is considerably greater than the difference in the Stokes forces acting on an isolated bubble or solid particle of the same size.

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